Exercise 334

For the following problems, consider the population of Ocean City, New Jersey, which is cyclical by season.

In reality, the overall population is most likely increasing or decreasing throughout each year. Let's reformulate the model as $P(t) = 82.5 - 67.5 \cos[(\pi/6)t] + t$, where t is time in months ($t = 0$ represents January 1) and P is population (in thousands). When is the first time the population reaches 200,000?

Solution

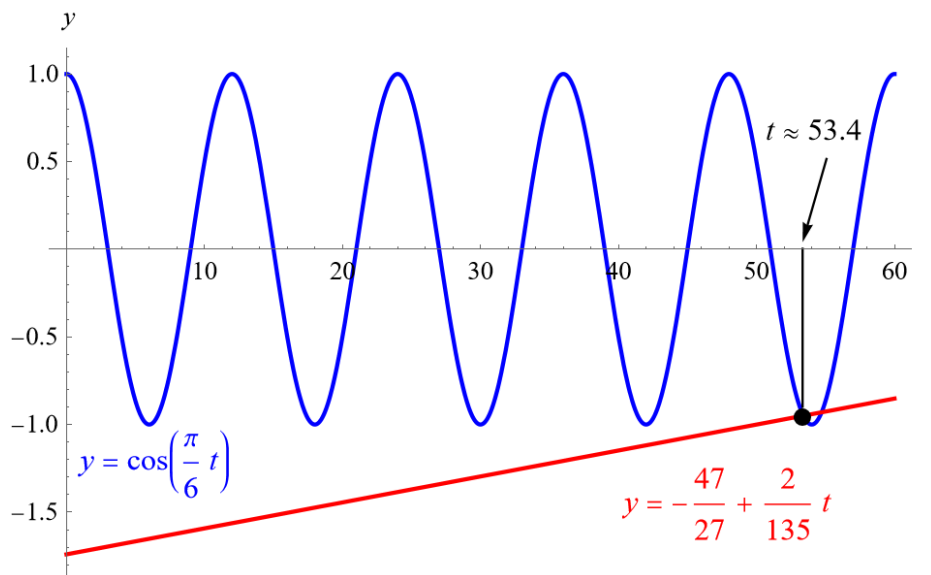Set $P(t) = 200$ and solve the equation for t.

$$P(t) = 82.5 - 67.5 \cos\left(\frac{\pi}{6}t\right) + t = 200$$

$$-67.5 \cos\left(\frac{\pi}{6}t\right) + t = 117.5$$

$$-67.5 \cos\left(\frac{\pi}{6}t\right) = 117.5 - t$$

$$\cos\left(\frac{\pi}{6}t\right) = -\frac{47}{27} + \frac{2}{135}t$$

Plot the function on the left versus t, and plot the function on the right versus t. The first time that they intersect is when the population first reaches 200,000.

The population first reaches 200,000 after about 53.4 months, or 4 years and 5 months and 11 days.

A plot of $P(t)$ versus t over the first 54 months is shown here, verifying the result above.